Dyn3 20.07.2011

③ $\forall \alpha \in so(3,\mathbb{R}) \hat{=}$ Antisimetrische 3×3 Matrizen $= \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$

$$A(t) = \exp(\alpha t)$$

$$G = \{A(t) \mid t \in \mathbb{R}\}$$

z.z.: $G \subset SO(3,\mathbb{R}) \hat{=} \boxed{\det A = 1}$, $A^{-1} = A^T$

und Gruppeneigenschaften:

i) $(A \circ B) \circ C = A \circ (B \circ C) \quad \forall A, \overset{e^{\alpha s}}{\overset{\shortparallel}{B}}, C \in G$

$(e^{\alpha r} \circ e^{\alpha s}) e^{\alpha t} = e^{\alpha r} \circ (\overset{e^{\alpha r}}{\overset{\shortparallel}{e^{\alpha s}}} \circ \overset{e^{\alpha t}}{\overset{\shortparallel}{e^{\alpha t}}})$ ✓

ii) E_3 neutr. Element:

$E_3 = e^{\alpha \cdot 0} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \underset{1}{0} \end{pmatrix}$ Einheitsmatrix ✓

$E_3 A = A$

(iii) Inverses:

$A^{-1} = e^{-\alpha r} = \exp\begin{pmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{pmatrix} \overset{!}{=} A^T$

$e^{-\alpha r} e^{\alpha r} = E_3$ ✓

$G \subset SO(3,\mathbb{R})$ dd • Gruppenoperation wie in $SO(3,\mathbb{R})$

$A^{-1} = A^T$ (iii)

und $\det(e^{\alpha r}) \underset{\text{Taylor}}{=} \det\Big(\underset{1}{\underset{\shortparallel}{id}} + \underbrace{\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & c & 0 \end{pmatrix}}r + \underbrace{\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & c & 0 \end{pmatrix}^2} r^2 \underbrace{\ldots}\Big)$

$\det(A)=0 \Rightarrow \det(A^n) = 0$

$= 1 + 0r + 0r + $ noch mehr 0en

$= 1$ ✓

Abgeschlossenheit: $e^{\alpha r} \circ e^{\alpha s} = e^{\alpha r + \alpha s} = e^{\alpha(r+s)}$ ✓

□