Dyn 3 — 08.05.2011

(11)

$$\dot{x} = f(x,\lambda) \qquad x \in \mathbb{R}^n$$
$$\dot{\lambda} = \sigma \qquad \lambda \in \mathbb{R}^m \qquad y := \begin{pmatrix} x \\ \lambda \end{pmatrix}$$

$$g(y) := \left(\begin{array}{c} f(x,\lambda) \\ \hline \sigma \end{array}\right) \begin{array}{l} \}\, n\times 1 \\ \}\, m\times 1 \end{array} \qquad f(0,\lambda) \equiv \sigma$$

$$D_y g = A = \left(\begin{array}{c|c} \overbrace{D_x f(0,\sigma)}^{=:B} & \sigma \\ \hline \sigma & \sigma \end{array}\right) \qquad A^T = \left(\begin{array}{c|c} B^T & 0 \\ \hline \sigma & 0 \end{array}\right)$$

$ad(B^T)$ Normalform von $\dot{x} = f(x,\sigma)$:

$$\hat{f}(x) = Bx + \hat{f}_2 + \dots$$
$$\hat{f}_\ell(x) = \sum_{|\sigma|=\ell}^{m} b_\sigma \cdot x_1^{\sigma_1} \cdot x_2^{\sigma_2} \cdots x_n^{\sigma_n}$$

$ad(A^T)$ Normalform von $\dot{y} = g(y)$:

$$\hat{g}(x,\lambda) = A\begin{pmatrix} x \\ \lambda \end{pmatrix} + \hat{g}_2 + \dots$$
$$\hat{g}_\ell(x,\lambda) = \sum_{|\sigma|=\ell} a_\sigma \cdot x_1^{\sigma_1} \cdot x_2^{\sigma_2} \cdots x_n^{\sigma_n} \cdot \lambda_1^{\sigma_{n+1}} \cdots \lambda_m^{\sigma_{n+m}}$$

<u>Es muss gelten:</u> $\tilde{T}_m \hat{g} \in W_\ell = \mathrm{Kern}(ad_m(A^T))$

$$ad_m(A^T)\, h^\sigma e_i = \sigma$$
$$\Leftrightarrow A^T h - h' A^T = \sigma \quad \Leftrightarrow A^T h = h' A^T$$

Bedingung für $\hat{f}_\ell$

$$\Leftrightarrow (D_x f(0,\sigma))^T \underset{\cap}{h} = h' B^T \quad \Rightarrow x\text{-Terme gleich denen in } \hat{f}_\ell$$

Polynome $\mathbb{R}^n$ da Komponenten $n+1$ bis $n+m$ eh auf σ abgebildet werden.

und $\sigma = \sigma \cdot \sum_{|\sigma|=\ell} a_\sigma x_1^{\sigma_1} x_2^{\sigma_2} \cdots x_n^{\sigma_n} \lambda_1^{\sigma_{n+1}} \cdots \lambda_m^{\sigma_{n+m}}$

$\sum_{|\sigma|=\ell} a_\sigma \dots$

$$\left(\begin{array}{c|c} B^T & \sigma \\ \hline \sigma & \sigma \end{array}\right)\left(\begin{array}{c} h_x \\ \hline h_\lambda \end{array}\right) = \left(\begin{array}{c|c} \partial_x h_x & \partial_\lambda h_x \\ \hline \partial_x h_\lambda & \partial_\lambda h_\lambda \end{array}\right)\left(\begin{array}{c|c} B^T & 0 \\ \hline 0 & 0 \end{array}\right) = \left(\begin{array}{c|c} \partial_x h_x B^T & \sigma \\ \hline \partial_x h_\lambda B^T & \sigma \end{array}\right)$$

$\Rightarrow$ λ kann als Parameter aufgefasst werden

$\Rightarrow$ definiere $b_\sigma = b_\sigma(\lambda) = \sum_{|\sigma_\lambda|=k} a_\sigma \lambda_1^{\sigma_{\lambda n+1}} \cdots \lambda_m^{\sigma_{\lambda m+n}}$

$\sigma_\lambda = \begin{pmatrix} \sigma_i & \text{für } i \geq n \\ \sigma & \text{sonst} \end{pmatrix}_i$

$$\Rightarrow \hat{f}_\ell(x,\lambda) = \sum_{|\sigma|=\ell+k} b_\sigma(\lambda)\, x_1^{\sigma_1} \cdots x_n^{\sigma_n}$$