Problem 14

z.z. $Ad(H_1) \circ Ad(H_2) = Ad(H_1 \circ H_2)$

$$\begin{aligned} Ad(H_1 \circ H_2) &= ((H_1 \circ H_2)^*)^{-1} \\ &= (H_2^* \circ H_1^*)^{-1} \\ &= (H_1^*)^{-1} \circ (H_2^*)^{-1} \\ &= Ad(H_1) \circ Ad(H_2) \end{aligned}$$

Linearität:

$$\begin{aligned} Ad(H)(g+\lambda f) &= H' \circ (g+\lambda f) \circ H^{-1} \\ &= H' \circ g \circ H^{-1} + \underline{\lambda H' \circ f \circ H^{-1}} \end{aligned}$$

(H' linear) ?

Inverses:

sei $(Ad(H))^{-1} := (H^{-1})' \circ g \circ H$

Überprüfen:

$$\underbrace{H'(H^{-1})'}_{Id} \circ g \circ \underbrace{H \circ H^{-1}}_{Id} = g \quad \checkmark$$

?

Aufgabe 15

$ad(h) . \mathfrak{g} \to \mathfrak{g}$

$$g \mapsto \frac{d}{dt}\Big|_{t=0} Ad(\exp(h \cdot t)) g \qquad H = ?$$

$$\begin{aligned} ?\ = {} & h \circ H(0) \circ g \circ H(0) - H(0) \circ g \circ h \circ H(0) \\ &= h \circ g - g \circ h \\ &= [h, g] \end{aligned}$$

Linearität:

$$\begin{aligned} [h, f + ag] &= h \circ (f+ag) - (f+ag) \circ h \\ &= h \circ f + a h \circ g - f \circ h - a g \circ h \\ &= [h,f] + a[h,g]. \end{aligned}$$

Hom of Lie algebras ?

$$\mathfrak{g} \longrightarrow GL(\mathfrak{g})$$

23.13 $\mathrm{ad}([g,h])f \overset{!}{=} [\mathrm{ad}\,g, \mathrm{ad}\,h]f$

$\mathrm{ad}\,h$: erhält $[\ ,\ ]$

$(\mathrm{ad}\,g\,\mathrm{ad}\,h - \mathrm{ad}\,h\,\mathrm{ad}\,g = \mathrm{ad}([g,h])$