Problem 40

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i) Schreibe $h_k := h(x_k) \in \mathbb{R}^n$

$\Rightarrow h_k \in \mathbb{R}^{nq} \quad \forall k$ $\quad (h_k)_{k\in\mathbb{Z}} \in X$

vielleicht $h: X \to X$, $(h(x))_k := h(x_k)$?

$= x_{k+2} - h(x_{k+1})$

$= x_{k+2} - h(x)_{k+1}$

$$
\begin{aligned}
(Sf(x))_k &= (f(x))_{k+1} \\
&= x_{k+2} - (Sh(x_k))_k \\
&= x_{k+2} - (Sh)_k \\
&= x_{k+2} - h_{k+1} \\
&= x_{k+2} - h(x_{k+1}) \\
&= f(x_{k+1}) \\
&= f((Sx)_k) .
\end{aligned}
$$

$f(Sx)_k = (Sx)_{k+1} - h((Sx)_k)$

$= x_{k+2} - h(x_{k+1})$

$\Rightarrow f$ ist equivariant bzgl $\mathbb{Z}_q$.

ii)
$$
\begin{aligned}
(SQx)_k &= (Qx)_{k+1} \\
&= \Big(\frac{1}{q}\sum_{j=0}^{q-1} \bar\mu_0^j \, \bar{Y}_{(r)}^T x_j\Big) \mu_0^{k+1} Y_{(r)} \\
&= \Big(\frac{1}{q}\sum_{j=0}^{q-1} \bar\mu_0^{j+1} \, \bar{Y}_{(r)}^T x_{j+1}\Big) \mu_0^{k+1} Y_{(r)} \\
&= \Big(\frac{1}{q}\sum_{j=0}^{q-1} \bar\mu_0^j \, \bar{Y}_{(r)}^T x_{j+1}\Big) \mu_0^{k} Y_{(r)} \\
&= Q(Sx)_k
\end{aligned}
$$

zunächst $\sum_{j=-1}^{q-2} \cdots$

wegen Index-Shift

aber $\mu_0^j = \mu_0^{j+q}$

wegen $\bar\mu_0 \mu_0 = 1$

$\Rightarrow$ die Projektion Q ist ebenfalls equivariant bzgl $\mathbb{Z}_q$.